Precision and Standard Deviation

Precision is the dispersion of, or closeness of the agreement between, successive measurements of the same quantity. The dispersion in a set of measurements is usually expressed in terms of the *standard deviation,* whose symbol is s:

$$s = \left(\frac{\sum d_i^2}{N-1}\right)^{1/2}$$

where

$\sum$ means "the sum of"

$d_i = x_i - \bar{x}$ = deviation

x_i = a particular value of a measurement

$\bar{x}$ = the mean value

N = the number of measurements

Measurements with high precision (Figure A.2A) are narrowly dispersed, and these measurements have a smaller standard deviation than measurements with lower precision (Figure A.2B).

Unless the number of measurements of the same quantity is very large, the calculated value of the standard deviation is only an estimate of the true standard deviation. Nevertheless, even a limited set of measurements will allow an estimate of the dispersion in the measurements to be judged.

How is the formula used? The formula states: Find the sum of the squares of the deviations, divide by one less than the total number of measurements, and take the square root of the result. The following example illustrates the use of the formula. Suppose we measure the length of an object seven times with a ruler. The values of these measurements (x_i) are 10.11 cm, 10.13 cm, 10.10 cm, 10.12 cm, 10.15 cm, 10.11 cm, and 10.12 cm. The calculation of the standard deviation of these results is shown in Table A.1.

Table A.1

An Example of Calculating a Standard Deviation

Value of Measurement x_i	Deviation $d_i = (x_i - \bar{x})$	Squared Deviation d_i^2
10.11	−0.01	0.0001
10.13	+0.01	0.0001
10.10	−0.02	0.0004
10.12	0.00	0.0000
10.15	+0.03	0.0009
10.11	−0.01	0.0001
10.12	0.00	0.0000
Sum = 70.84		Sum = 0.0016

$$\text{Mean } (\bar{x}) = \frac{\Sigma x_i}{N} = \frac{70.84}{7} = 10.12$$

$$s = \left(\frac{0.0016}{7-1}\right)^{1/2} = 0.016$$

You can obtain an identical result using the tool available online at the student website. Standard deviation materials can be found at

http://college.hmco.com/PIC/ebbing9e

When a quantity, such as the length of an object, is measured several times, it is customary to report the mean value of the measurements. The dispersion, or precision, of the measurements can be indicated, according to one custom, by writing ± the calculated value of s after the mean. Thus we would report 10.12 ± 0.02 cm for the example in Table A.1. Note that the standard deviation of 0.016 was rounded to two figures because there are only two figures to the right of the decimal point in the mean. When we report 10.12 ± 0.02 cm as the best value for the quantity, we are stating that the length of the object probably lies between

$$10.12 + 0.02 = 10.14 \text{ cm}$$

and

$$10.12 - 0.02 = 10.10 \text{ cm}$$

Precision and Significant Figures

The precision of a set of measurements can also be gauged by the number of significant (that is, meaningful) figures that are used in the mean value of the measurements. This is the method that is required in the experiment "Some Measurements of Mass and Volume."

The correct number of significant figures in a mean value will always be the number of certain digits plus one uncertain digit. In the example in Table A.1, we have shown that the length of the object probably lies between 10.10 cm and 10.14 cm, with a mean value of 10.12 cm. Clearly, the first three digits in the mean value are certain, and uncertainty occurs in the fourth digit. The precision of these measurements justifies the use of four significant figures in the mean value.